

Radial cancellation in spinning sound fields

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Abstract

The radiating part of a circular acoustic source is determined, on the basis of an exact analysis of the radiation properties of a source with angular dependence $\exp jn\theta$ and arbitrary radial dependence. It is found that the number of degrees of freedom in the radiated field is no greater than $k-n$, where k is the wavenumber. The radiating part of the source at low frequency is explicitly stated and used to analyze noise cancellation. The results are applied to the identification of sources in jet noise and an explanation for the low order structure of jet noise fields is proposed.

1 Introduction

Sound generation by spinning modes is a central problem in many applications. Devices such as propellers and fans obviously produce a rotating source system, but the termination of a circular duct also radiates like a spinning source and jets have a source system which can be decomposed into spinning modes. The problem of the relationship between a source distribution and its spinning acoustic field has thus attracted considerable attention in the literature.

This paper examines one part of the problem, the relationship between the radial structure of the source and the form of the corresponding acoustic field. Earlier work, using exact and asymptotic analysis (Carley, 2010*a,b*), has fixed an upper limit on the information which is radiated into the field, but without considering the effect of the source distribution. In this paper, the theory is extended to include the relationship between the source and the field, allowing a discussion of the implications for a number of problems.

A first area where the analysis is relevant is that of control of noise from rotors. One approach to this problem is to fit an inverse model to the measured noise and then use this model to compute a noise field which cancels the rotor noise at some point. In a recent study (Gérard *et al.*, 2005*a,b*, 2007) it was found that a very low order acoustic model of a cooling fan was sufficiently accurate for control purposes. At first glance, it is not clear why a low order model should give a good match to the results from a finite rotor, beyond considerations of acoustical compactness. An analysis of the information content of the field, however, shows that the field is generated by a set of low order modes with

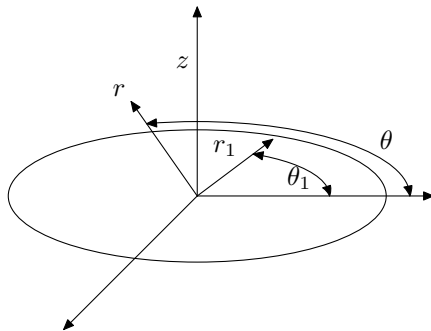


Figure 1: Coordinate system for disc radiation calculations

the higher order modes being cut-off and generating exponentially small noise (Carley, 2010*a,b*).

The second application of the approach of this paper is to inverse methods, in which a ‘source’ is determined from acoustic field measurements. There have been numerous applications of such techniques, but of particular interest here is that of jet noise. It is known that the noise field of a turbulent jet is represented by a much lower order model than is the flow (Jordan *et al.*, 2007). This is partly explained by axial interference effects (Freund, 2001, for example) but relatively little attention has been paid to the influence of the radial structure of the source until quite recently (Michel, 2009).

This paper presents an analysis of the general problem of radiation from a disc source and fixes limits on the proportion of the source which actually radiates into the acoustic field, with no approximation other than the standard acoustic assumption of linearity. In particular, the far-field assumption is not required, making the results applicable over a wide range of parameters. The implications of the results are then discussed with respect to rotor and jet noise.

2 Spinning sound fields

The problem is formulated as that of calculating the acoustic field radiated by a monopole source distributed over a circular disc. This disc may be viewed as the source proper, such as in the case of rotor noise, or as part of a distributed three-dimensional source, as in jet noise. The system for the analysis is shown in Figure 1 with cylindrical coordinates (r, θ, z) . All lengths are non-dimensionalized on disc radius and the disc lies in the plane $z = 0$. The field from one azimuthal mode of the acoustic source, specified as $s_n(r_1) \exp jn\theta_1$, has the form $P_n(k, r, z) \exp jn\theta$, with P_n given by the Rayleigh integral:

$$P_n(k, r, z) = \int_0^1 \int_0^{2\pi} \frac{e^{j(kR' + n\theta_1)}}{4\pi R'} d\theta_1 s_n(r_1) r_1 dr_1, \quad (1)$$

$$R' = [r^2 + r_1^2 - 2rr_1 \cos \theta_1 + z^2]^{1/2},$$

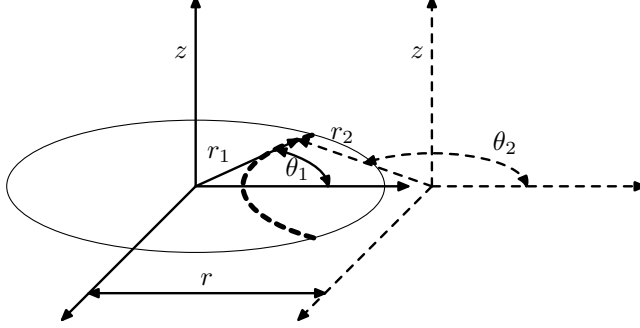


Figure 2: Transformation to equivalent line source

where k is wavenumber and subscript 1 indicates variables of integration. The field due to higher order sources, such as dipoles and quadrupoles, would be found by differentiation of (1). The analysis to be presented does not include such sources but the conclusions drawn should still be valid.

2.1 Equivalent line source expansion

The analysis of the nature of the sound field from an arbitrary disc source is based on a transformation of the disc to an exactly equivalent line source, an approach which has been used to study transient radiation from pistons (Oberhettinger, 1961; Pierce, 1989), rotor noise (Chapman, 1993; Carley, 1999) and source identification methods (Carley, 2009).

The transformation to a line source is shown in Figure 2, which shows the new coordinate system (r_2, θ_2, z) centred on a sideline of constant radius r . Under this transformation:

$$P_n(r, z) = \int_{r-1}^{r+1} \frac{e^{jkR'}}{R'} K(r, r_2) r_2 dr_2, \quad (2)$$

$$R' = (r_2^2 + z^2)^{1/2},$$

$$K(r, r_2) = \frac{1}{4\pi} \int_{\theta_2^{(0)}}^{2\pi - \theta_2^{(0)}} e^{jn\theta_1} s_n(r_1) d\theta_2, \quad (3)$$

for observer positions with $r > 1$, with the limits of integration given by:

$$\theta_2^{(0)} = \cos^{-1} \frac{1 - r^2 - r_2^2}{2rr_2}. \quad (4)$$

Functions of the form of $K(r, r_2)$ have been analyzed in previous work (Carley, 1999). In this paper, it is sufficient to note that the source function can be written (Carley, 2009):

$$K(r, r_2) = \sum_{q=0}^{\infty} u_q(r) U_q(s) (1 - s^2)^{1/2}, \quad (5)$$

where $U_q(s)$ is a Chebyshev polynomial of the second kind, $s = r_2 - r$ and the coefficients $u_q(r)$ are functions of r but not of z . Inserting (5) into (2):

$$P_n(k, r, z) = \sum_{q=0}^{\infty} u_q(r) \int_{-1}^1 \frac{e^{jkR'}}{R'} U_q(s) (r+s)(1-s^2)^{1/2} ds, \quad (6)$$

$$R' = [(r+s)^2 + z^2]^{1/2}. \quad (7)$$

The radiation properties of the integrals of (6) have been examined in some detail elsewhere (Carley, 2010a,b), giving an exact result for the in-plane case $z = 0$:

$$P_n(k, r, 0) = \pi e^{jkr} \sum_{q=0}^{\infty} u_q(r) j^q (q+1) \frac{J_{q+1}(k)}{k}. \quad (8)$$

For large order q , the Bessel function $J_q(k)$ decays exponentially for $k < q$ so that the line source modes with order $q > k$ are ‘cut-off’ and generate exponentially small noise fields. Since the integrals have their maximum in the plane $z = 0$, (8) says that the whole field is exponentially small and modes $u_q(r)$ with $q > k$ are cut-off everywhere. This gives an indication of how much of a given source distribution radiates into the acoustic field, near or far. There only remains to establish the relationship between the radial source $s_n(r_1)$ and the line source coefficients $u_q(r)$.

2.2 Series expansion for spinning sound fields

A recently derived series (Carley, 2010c) for the field radiated by a ring source of radius r_1 can be used to find an expression for the sound radiated by a disc source with arbitrary radial variation:

$$\begin{aligned} R_n &= \int_0^{2\pi} \frac{e^{j(kR' + n\theta_1)}}{4\pi R'} d\theta_1, \\ &= j^{2n+1} \frac{\pi}{4} \frac{1}{(r_1 R)^{1/2}} \sum_{m=0}^{\infty} (-1)^m \frac{(2n+4m+1)(2m-1)!!}{(2n+2m)!!} \\ &\times H_{n+2m+1/2}^{(1)}(kR) P_{n+2m}^n(\cos \phi) J_{n+2m+1/2}(kr_1), \end{aligned} \quad (9)$$

with $H_{\nu}^{(1)}(x)$ the Hankel function of the first kind of order ν , J_{ν} the Bessel function of the first kind and P_n^m the associated Legendre function. The observer position is specified in spherical polar coordinates $R = [r^2 + z^2]^{1/2}$, $\phi = \tan^{-1} r/z$.

Multiplication by the radial source term $r_1 s_n(r_1)$ and integration gives an expression for the field radiated by a general source of unit radius and azimuthal order n :

$$\begin{aligned} P_n(k, r, z) &= j^{2n+1} \frac{\pi}{4} \sum_{m=0}^{\infty} (-1)^m \frac{(2n+4m+1)(2m-1)!!}{(2n+2m)!!} P_{n+2m}^n(\cos \phi) S_{n+2m}, \\ S_{n+2m}(k, r, z) &= \int_0^1 s_n(r_1) J_{n+2m+1/2}(kr_1) H_{n+2m+1/2}^{(1)}(kR) \left(\frac{r_1}{R}\right)^{1/2} dr_1. \end{aligned}$$

Setting $z = 0$ ($\phi = \pi/2$, $R = r$):

$$P_n(k, r, 0) = \frac{j\pi}{4} \sum_{m=0}^{\infty} \frac{1}{m!} \frac{(2n+4m+1)(2n+2m-1)!!(2m-1)!!}{2^m(2n+2m)!!} S_{n+2m} \quad (10)$$

where use has been made of the expression (Gradshteyn & Ryzhik, 1980, 8.756.1):

$$P_{n+2m}^n(0) = \frac{(-1)^{m+n}}{2^m} \frac{(2n+2m-1)!!}{m!}. \quad (11)$$

2.3 Line source coefficients

The expressions for P_n from § 2.2 and § 2.1 are both exact and can be equated to derive a system of equations relating the coefficients $u_q(r)$ to the radial source distribution $s_n(r_1)$:

$$\begin{aligned} \frac{j}{4} \sum_{m=0}^{\infty} \frac{1}{m!} \frac{(2n+4m+1)(2n+2m-1)!!(2m-1)!!}{2^m(2n+2m)!!} S_{n+2m} = \\ e^{jkr} \sum_{q=0}^{\infty} u_q(r) j^q (q+1) \frac{J_{q+1}(k)}{k}. \end{aligned} \quad (12)$$

Under repeated differentiation, (12) becomes a lower triangular system of linear equations which connects the coefficients $u_q(r)$ and S_{n+2m} :

$$\begin{aligned} \frac{j}{4} \sum_{m=0}^{\infty} \frac{1}{m!} \frac{(2n+4m+1)(2n+2m-1)!!(2m-1)!!}{2^m(2n+2m)!!} S_{n+2m}^{(v)} = \\ \sum_{q=0}^{\infty} u_q(r) j^q (q+1) \left[e^{jkr} \frac{J_{q+1}(k)}{k} \right]^{(v)}, \end{aligned} \quad (13)$$

where superscript (v) denotes the v th partial derivative with respect to k , evaluated at $k = 0$.

Using standard series (Gradshteyn & Ryzhik, 1980), the products of special functions can be written:

$$\begin{aligned} e^{jkr} \frac{J_{q+1}(k)}{k} &= \frac{1}{j^q} \sum_{t=0}^{\infty} (jk)^{t+q} E_{t,q}(r), \\ E_{t,q}(r) &= \frac{1}{2^{q+1}} \sum_{s=0}^{[t/2]} \frac{r^{t-2s}}{4^s s! (s+q+1)! (t-2s)!}, \end{aligned} \quad (14)$$

where $[t/2]$ is the largest integer less than or equal to $t/2$, and

$$\begin{aligned} \left(\frac{r_1}{r}\right)^{1/2} H_{n+1/2}^{(1)}(kr) J_{n+1/2}(kr_1) &= \left(\frac{r}{2}\right)^{2n+1} \sum_{t=0}^{\infty} \frac{k^{2t+2n+1}}{t!} \left(-\frac{r^2}{4}\right)^t V_{n,t}(r_1/r) \\ &- (-1)^n j \sum_{t=0}^{\infty} \frac{k^{2t}}{t!} \left(-\frac{r^2}{4}\right)^t W_{n,t}(r_1/r), \end{aligned} \quad (15)$$

with the polynomials $V_{n,t}$ and $W_{n,t}$ given by:

$$V_{n,t}(x) = \sum_{s=0}^t \binom{t}{s} \frac{x^{2s+n+1}}{\Gamma(n+s+3/2)\Gamma(t-s+n+3/2)}, \quad (16a)$$

$$W_{n,t}(x) = \sum_{s=0}^t \binom{t}{s} \frac{x^{2s+n+1}}{\Gamma(n+s+3/2)\Gamma(t-s-n+1/2)}. \quad (16b)$$

Given the power series, the derivatives at $k=0$ are readily found:

$$\begin{aligned} \left. j^q \frac{\partial^v}{\partial k^v} e^{jkr} \frac{J_{q+1}(k)}{k} \right|_{k=0} &= \begin{cases} 0, & v < q, \\ j^v v! E_{v-q,q}(r), & v \geq q. \end{cases} \quad (17a) \\ \left. \frac{\partial^v}{\partial k^v} (r_1/r)^{1/2} H_{n+1/2}^{(1)}(kr) J_{n+1/2}(kr_1) \right|_{k=0} &= \\ \begin{cases} 0, & v = 2v' + 1, \quad v' < n; \\ \left(\frac{r}{2}\right)^{2n+1} \left(-\frac{r^2}{4}\right)^{v'-n} \frac{v!}{(v'-n)!} V_{n,v'-n}(r_1/r), & v = 2v' + 1, \quad v' \geq n; \\ -(-1)^n j \frac{(2v')!}{v'!} \left(-\frac{r^2}{4}\right)^{v'} W_{n,v'}(r_1/r), & v = 2v'. \end{cases} \quad (17b) \end{aligned}$$

Setting $v = 0, 1, \dots$ yields an infinite lower triangular system of equations for $u_q(r)$:

$$\mathbf{E}\mathbf{U} = \mathbf{B}, \quad (18)$$

with $\mathbf{U} = [u_0 \ u_1 \ \dots]^T$ and the elements of matrix \mathbf{E} and vector \mathbf{B} given by:

$$E_{vq} = \begin{cases} j^v (q+1) v! E_{v-q,q}(r), & q \leq v; \\ 0, & q > v. \end{cases} \quad (19a)$$

$$B_v = \frac{j}{4} \sum_{m=0}^{\infty} \frac{1}{m!} \frac{(2n+4m+1)(2n+2m-1)!!(2m-1)!!}{2^m(2n+2m)!!} S_{n+2m}^{(v)}, \quad (19b)$$

where

$$S_{n+2m}^{(v)} = \begin{cases} 0 & v = 2v' + 1, \quad v' < n + 2m; \\ (-1)^{n+v'} \frac{v!}{(v'-n-2m)!} \left(\frac{r}{2}\right)^v \int_0^1 V_{n+2m,v'-n-2m}(r_1/r) s_n(r_1) dr_1, & v = 2v' + 1, \quad v' \geq n + 2m; \\ -(-1)^{n+v'} \frac{jv!}{v'!} \left(\frac{r}{2}\right)^v \int_0^1 W_{n+2m,v'}(r_1/r) s_n(r_1) dr_1, & v = 2v'. \end{cases}$$

Given a radial source term $s_n(r_1)$, (18) can be solved to find the coefficients $u_q(r)$ of the equivalent line source modes. Since it is lower triangular, the first few values of u_q can be reliably estimated, although ill-conditioning prevents accurate solution for arbitrary large q .

2.4 Radiated field

From the relationship between the radial source term and the line source coefficients, some general properties of the acoustic field can be stated. The first result, already found by Carley (2010*a,b*) is that, since the line source modes with $q + 1 > k$ are cut off, the acoustic field has no more than k degrees of freedom, in the sense that the radiated field is given by a weighted sum of the fields due to no more than k elementary sources. From (18), this result can be extended.

The first extension comes from the fact that $B_{2v+1} \equiv 0$, for $v' < n$, on the right hand side of (18). This means that u_q , $q = 2v' + 1$, is uniquely defined by the lower order coefficients with $q \leq 2v'$. The result is that the acoustic field of azimuthal order n , whatever might be its radial structure, has no more than $k - n$ degrees of freedom.

A second extension comes from examination of (18) The first few entries of the system of equations are:

$$\begin{bmatrix} 1/2 & 0 & 0 & 0 & \cdots \\ r/2 & 1/4 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & 0 & \cdots \end{bmatrix} \begin{pmatrix} u_0 \\ u_1 \\ \vdots \end{pmatrix} = \begin{pmatrix} B_0 \\ 0 \\ \vdots \end{pmatrix}, \quad (21)$$

resulting in the solution:

$$u_0 = 2B_0; \quad u_1 = -2ru_0 = -4rB_0, \quad (22)$$

so that the ratio of u_0 and u_1 is constant, for arbitrary $s_n(r_1)$. This means that low frequency sources of the same radius and azimuthal order generate fields which vary only by a scaling factor, since the higher order terms are cut off.

Finally, if we attempt to isolate a source $s_n(r_1)$ associated with a single line source mode, by setting $u_q \equiv 1$ for some q , with all other $u_q \equiv 0$, we find that the line modes must occur in pairs, since if $u_{2v'} \equiv 1$, $u_{2v'+1} \neq 0$, being fixed by the condition $B_{2v'+1} \equiv 0$.

3 Results

To illustrate the application of the result of the previous section, we present some results for the calculation of the line source coefficients and for the use of the method to modify the radiating part of a source. We also discuss qualitatively the implications of the results for studies of jet noise.

3.1 Line source coefficient evaluation

The first results are a comparison of coefficients $u_q(r)$ computed using (18) and those computed directly from exact closed-form expressions for $K(r, r_2)$ in the case when the radial source term is a monomial in radius $s_n = r_1^a$ (Carley, 1999). Figure 3 compares the two sets of coefficients for $a = 0, 2, 4$, with the

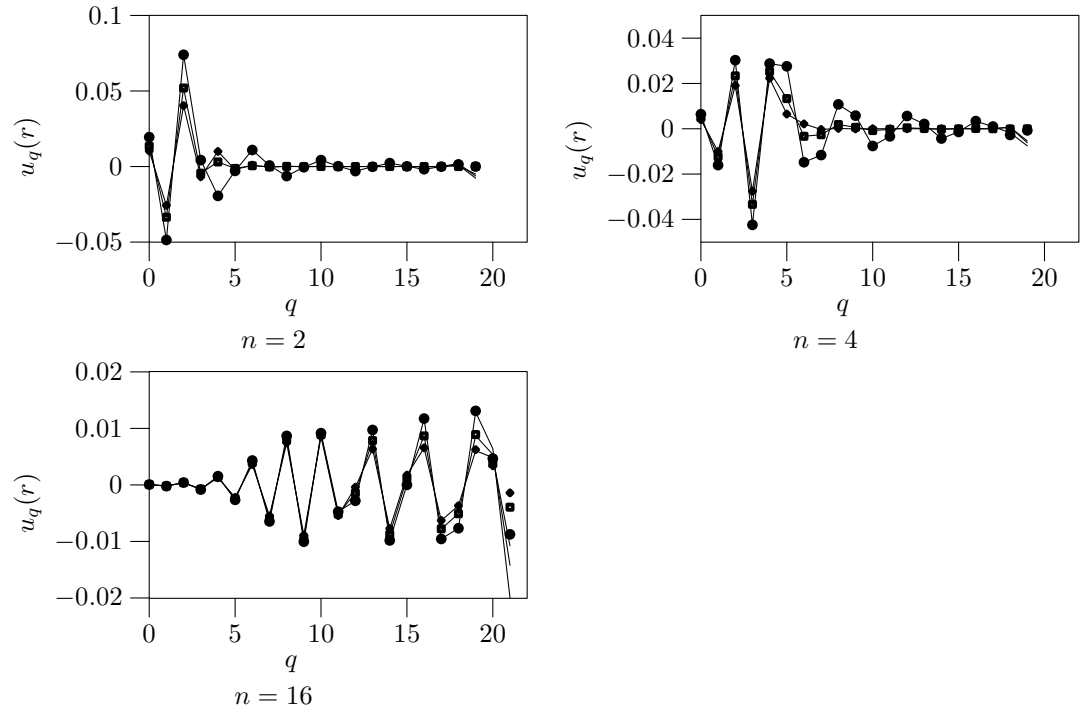


Figure 3: Line source mode coefficients computed using the method of § 2.3 (solid lines) and directly from analytical formulae (symbols) for $r = 5/4$, $s = r_1^a$, $a = 0$ (circles), $a = 2$ (squares) and $a = 4$ (diamonds) for $n = 2, 4, 16$.

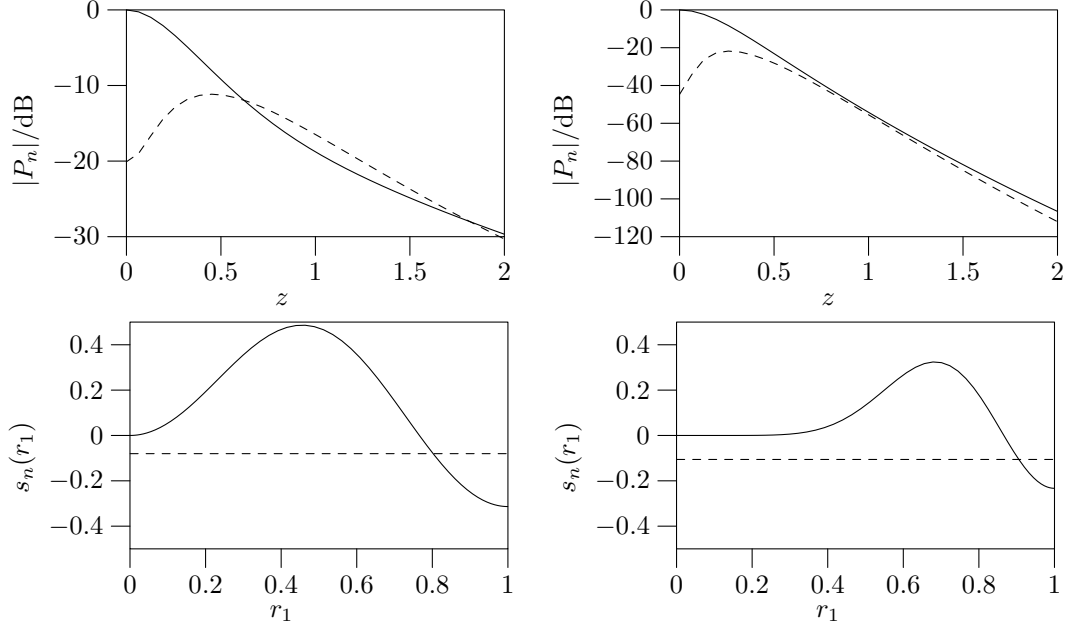


Figure 4: Cancellation effects for radial source terms: top row radiated field from original $s_n(r_1)$ (solid) and modified source $s_n(r_1) - \beta s'_n(r_1)$ (dashed); bottom row source terms $s_n(r_1)$ (solid) and $\beta s'_n(r_1)$ dashed. The left hand column covers the case $n = 2$ and the right hand $n = 8$. In both cases, $k = 1$, $r = 5/4$.

plots terminated at a value of q where the difference between the two sets of results becomes noticeable, $q \approx 20$. This gives an indication of the effect of the ill conditioning of (18). For $q \lesssim 20$, the computed values of u_q are reliable. It is noteworthy that for small q , the coefficients are practically equal for all values of a so that for low frequency radiation, the radiated fields will be practically indistinguishable.

3.2 Radial cancellation

The analysis so far has identified that part of a source distribution which radiates, as a function of wavenumber k . From the results, it appears that only a small part of the source is responsible for the acoustic field, with most of the line source modes being cut off throughout the field, except at high frequency. Indeed, asymptotic analysis (Carley, 2010a,b) shows that even the cut-on modes radiate efficiently only into a small part of the acoustic field, with the exception of those of low order. In any case, this offers a method for examination of the radiation properties of a source. Given a source term $s_n(r_1)$ the approach is to impose a secondary source $s'_n(r_1)$ which generates the same set of line source modes up to some required order. In the simplest case, we match the first line

source coefficient u_0 , which automatically matches u_1 . If u_0 is known for both $s_n(r_1)$ and for $s'_n(r_1)$, then the source $s'_n(r_1) - \beta s_n(r_1)$ will have $u_0 = u_1 = 0$, if β is taken as the ratio of u_0 for the primary and secondary sources $s_n(r_1)$ and $s'_n(r_1)$.

Figure 4 shows the results of such a procedure using sources $s_n(r_1) = J_n(\alpha r_1)$, with α the first extremum of J_n (similar to a duct mode), and $s'_n(r_1) \equiv 1$. The wavenumber $k = 1$ and values $n = 2, 8$ have been used. In the first case, $n = 2$, the noise reduction in the plane $z = 0$ is quite large, about 20dB, but there is a slight increase around $z = 1$. This is because, as seen in Figure 3, the coefficient u_2 is quite large and is multiplied by a Bessel function of order 3, which is not of high enough order for the exponential decay with k which cuts off the mode.

The cut-off behaviour is seen more clearly in the $n = 8$ case, where the reduction at $z = 0$ is 40dB. At larger z , the reduction is much smaller, but this is because, as found from asymptotic analysis (Carley, 2010*a,b*), the field in this region only contains contributions from the remaining lower order modes, starting with $q = 2$.

3.3 Degrees of freedom in jet noise fields

The results of § 2.4, regarding the number of degrees of freedom in the acoustic field, can help explain some features of experiments on jet noise. Despite the lack of consensus on what is meant by the ‘source’ of jet noise (Jordan & Gervais, 2008; Suzuki, 2010), some progress has been made by assuming that the source of jet noise can be identified with some combination of flow quantities. An open question, however, is which part of the source term radiates, since it is clear that only a small fraction of the flow generates the acoustic field. Two recent sets of results, one experimental, the other numerical, illustrate the issues.

In one, Freund (2001) has used direct numerical simulation to compute the flow and noise of a Mach 0.9 jet, validating the noise prediction against experiments and showing that a Lighthill (1952) source term accurately reproduces the acoustic field. Spatial filtering of the source, using a wavenumber criterion to remove the non-radiating part, left “a set of modes capable of radiating to the far field”, with the caveat that “additional cancellation may occur due to the radial structure of the source which is not accounted for in this analysis”. Indeed, the radial structure of jet noise sources has not received much attention until quite recently (Michel, 2009).

An experimental result of some interest is that of Jordan *et al.* (2007) who performed a modal decomposition of a jet flow field and a proper orthogonal decomposition optimized for the resolution of the far field noise. They found that more than 350 modes were needed to capture half of the flow energy while 24 modes sufficed for 90% of the far-field noise. As they note, passage to the far field acts as a filter passing only a low-dimensional representation of the flow.

From these observations, it is plausible that the relatively low order structure of jet noise can be explained by the results of this paper. In the notation of this paper, $k = \pi StM$, where St is Strouhal number based on jet diameter and M

is jet Mach number. For the range of Strouhal number important for jet noise $St < 2$ (Michalke & Fuchs, 1975), $k < 2\pi M$. For the $M = 0.9$ jet studied by Jordan *et al.* (2007), for example, this yields $k \lesssim 5.7$ and no more than about six line source modes radiate from the axisymmetric source modes at the highest frequency of interest. This estimate would need to be modified to take account of axial interference as in the far field analysis of Michel (2009) but does offer the possibility of establishing some reasonable limits on the detail to be expected from acoustic measurements on jets and the requirements for low order models used in noise control.

4 Conclusions

The radiation properties of disc sources of arbitrary radial variation have been analyzed to establish the part of the source which radiates into the acoustic field, without recourse to a far field approximation. Limits have been established on the number of degrees of freedom of the part of the source which radiates and the implications of these limits have been discussed for the problems of rotor noise and studies of source mechanisms in jets. Future work will consider the use of the findings of this paper to study the radiating portion of full jet source distributions, including axial interference effects.

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